SOLVING FUZZY MULTI-OBJECTIVE NONLINEAR FRACTIONAL PROGRAMMING PROBLEM USING FUZZY PROGRAMMING MODEL

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Abstract: The aim of this paper is to present a method, in which a fuzzy multi-objective nonlinear fractional programming problem is presented. All the coefficients of the nonlinear fractional multi-objective functions and the constraints are taken to be fuzzy numbers and using ranking function. The method has been illustrated with numerical example.

Keywords: Fuzzy Multi-objective Nonlinear fractional programming problem, multi-objective nonlinear fractional programming problem, trapezoidal fuzzy numbers, crisp problem, Ranking function.

1. INTRODUCTION

Several factors in the real world imply the increase in use of nonlinear programming models. There are several classes of problems, where nonlinear programming had had a great impact, for example oil and petrochemical industries, nonlinear network problems and economic planning models [1]. The area where nonlinear programming can be used in these several classes of problems is given in detail in frequently, we may face decision to optimize department/equity ratio, profit /cost, inventory/sales, actual cost/standard cost, output, etc. With respect to some constraints , this type of problem is referred to as fractional programming problem. Most of the real world's problems of above category are characterized by multiple conflicting and incommensurate aspects of evaluation. These problems are optimized in the framework of multiple objective fractional programming models.

Furthermore, while addressing some real world problems, frequently the parameters are imprecise numerical quantities. Fuzzy quantities are very adequate for modeling this situation. Under this situation, the above category of problems is termed as multi-objective fuzzy fractional programming problems (MOFFPP). Bellman and Zadeh [2] introduced the concept of fuzzy quantities and also the concept of fuzzy decision making. H.J.Zimmermann [15] has introduced fuzzy programming approach to solve crisp multi-objective linear programming problem. A.Mounz [4], Roubens [9] reduced fuzzy multi-objective linear programming to crisp problems using Ranking function. Methods based on comparison of fuzzy numbers have been suggested by H.R.Maleki [12], A.Ebrahimnejad, S.H.Nasseri [8], F.Roubens [9] and L.Campos et.[3]. Zimmermann [15] has introduced fuzzy programming approach to solve crisp multi-objective linear programming problem. Recently H.M.Nehi et.al., [13] used ranking function suggested by M.Delgaodo et.al [5] to solve fuzzy MOLPP.Kanti Swarup [10] gave an algorithm for the solution of LFPP without reducing it to LPP.

In this paper, we develop a method for solving multi-objective nonlinear fuzzy fractional programming problem, where all the parameters of the objective functions and the constraints are fuzzy in nature. In this method, firstly the MOFFPP is reduced to MOFNLPP. Similarly, the MOFNLPP is reduced to MONLPP using the Ranking function of Rouben's [9], then the MOLPP is solved by using fuzzy programming approach of Zimmermann [15], [16]. A numerical example is given at the end to illustrate the method of solution.

II.NONLINEAR FRACTIONAL PROGRAMMING PROBLEM

A linear fractional programming problem is given by

$$\text{Max/min} \quad z = \frac{\sum_{j=1}^{n} c_{rj} x_j^{\delta j} + \alpha}{\sum_{j=1}^{n} d_{rj} x_j^{\delta j} + \beta} \qquad r= 1, 2, \ldots .q$$

Subject to $\sum_{j=1}^{n} a_{ij} x_j \le b_i$, i= 1, 2, ... m

Definition: 2.1

X* is said to be a complete optimal solution for (1) if there exist $x^* \in X$ such that

$c_i(x^*) \ge c_i(x)$, i = 1, 2 . . . k for all $x \in X$.

III. RANKING FUNCTION FOR FUZZY NUMBERS

Definition: 3.1

Let A be a fuzzy number whose membership function can generally be defined as

$$\mu_A(x) = \begin{cases} \mu_A^L(x) & a^1 \le x \le a^2 \\ 1 & a^2 \le x \le a^3 \\ \mu_A^R(x) & a^3 \le x \le a^4 \\ 0 & \text{Otherwise} \end{cases}$$

Where $\mu_A^L(x) : [a^1, a^2] \to [0, 1]$ and $\mu_A^R(x) : [a^3, a^4]$ are strictly monotonic and continuous mappings. Then, it is considered as left right fuzzy number. If the membership function $\mu_A(x)$ is piecewise linear, then it is referred to as a trapezoidal fuzzy number and is usually denoted by A= (a^1, a^2, a^3, a^4). If $a^2 = a^3$, the trapezoidal fuzzy number is turned into a triangular fuzzy number A = (a^1, a^3, a^4).

A fuzzy number A = (a, b, c) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ 1 & x = b \\ \frac{x-b}{b-c} & b \le x \le c \\ 0 & \text{Otherwise} \end{cases}$$

Assume that R : F(R) → R be linear ordered function that maps each fuzzy number in to the real number, in which F(R) denotes the whole fuzzy numbers. Accordingly, for any two fuzzy numbers $\tilde{a}$ and $\tilde{b}$, we have

$$\tilde{a} \underset{R}{\ge} \tilde{b} \text{ iff } R(\tilde{a}) \ge R(\tilde{b})$$

$$\tilde{a} \underset{R}{\succ} \tilde{b} \text{ iff } R(\tilde{a}) > R(\tilde{b})$$

$$\tilde{a} \underset{R}{=} \tilde{b} \text{ iff } R(\tilde{a}) = R(\tilde{b})$$

We restrict our attention to linear ranking function R such that R $(k\tilde{a} + \tilde{b})$ = k $R(\tilde{a})$ + R $(\tilde{b})$ for any $\tilde{a}$ and $\tilde{b}$ in F(R) and any $k \in R$.

Roubens' Ranking Function:

The ranking function proposed by F. Roubens is defined by

$$R(\tilde{a}) = 1/2 \int_0^1 (\inf \tilde{a}_\alpha + \sup \tilde{a}_\alpha)\, d\alpha$$

Which reduces to

$$R(\tilde{a}) = ½ (a^L + a^U + ½ (\beta - \alpha)). \quad . . . . . (3.1)$$

For a trapezoidal number

$$\tilde{a} = (a^L - \alpha , a^L, a^U, a^U + \beta)$$

IV.SOLVING FUZZY MULTI-OBJECTIVE NONLINEAR FRACTIONAL PROGRAMMING PROBLEM

A fuzzy multi-objective linear fractional programming problem is defined as follows [4, 8]

$$\text{Max/min } \tilde{z} = (\tilde{z}_1, \tilde{z}_2 . . . . .\tilde{z}_p)$$

$$\text{Where } \tilde{z}_j = \frac{\tilde{c}_{rj}^T + \tilde{\alpha}_j}{\tilde{d}_{rj}^T + \tilde{\beta}_j} \quad j = 1,2, ... p \quad . . . . (4.1)$$

Subject to $\tilde{A}_i\, x \le = \ge \tilde{b}_i$, $x \ge 0$
i= 1,2,. . . .m

Where $\tilde{c}_{rj}, \tilde{d}_{rj}$ are (n X 1) fuzzy vectors,

$\tilde{A}_i$ are (m X n) matrices on thefuzzy numbers
$\tilde{b}_i$ are (m X 1) fuzzy vectors
$\tilde{\alpha}_j$ and $\tilde{\beta}_j$ are fuzzy numbers.

Step-1:

The fuzzy multi-objective nonlinear fractional programming problem (4.1) is first reduced to multi-objective fuzzy nonlinear programming problem

Max/ min $\tilde{y} = (\tilde{y}_1, \tilde{y}_2, \dots \dots . . \tilde{y}_p)$
Subject to $\tilde{A}_i\, x \le = \ge \tilde{b}_i$, $x \ge 0$

Where $\tilde{y}_j = (\tilde{c}_{rj}^T + \tilde{\alpha}_j) - (\tilde{d}_{rj}^T + \tilde{\beta}_j)$
r = 1,2,. . . . . q

It is reduced to the form $\tilde{y}_j = \sum_{i=1}^{n} \tilde{e}_{ij} x_j^{\delta_j} + \tilde{\gamma}_i$

Thus, the problem (4.1) is reduced to the multi-objective nonlinear programming problem as follows:

Max/ min $\widetilde{Y}$ =

$$\sum_{j=1}^{n}\widetilde{e}_{1j}x_j^{\delta_j}+\widetilde{\gamma}_1,\sum_{j=1}^{n}\widetilde{e}_{2j}x_j^{\delta_j}+\gamma_2,\ldots\ldots.\sum_{j=1}^{n}\widetilde{e}_{pj}x_j^{\delta_j}+\widetilde{\gamma}_p)$$

. . .. . .(4.2)

Subject to $\sum_k \widetilde{a}_{jk}^i x_k \le \widetilde{b}_j^i, x_k \ge 0$

In trapezoidal form

$\widetilde{e}_{ij} = (e_{ij}^1, e_{ij}^2, e_{ij}^3, e_{ij}^4)$

$\widetilde{a}_{jk}^i = (a_{jk}^{i1}, a_{jk}^{i2}, a_{jk}^{i3}, a_{jk}^{i4})$

$\widetilde{\gamma}_i = (\widetilde{\gamma}_i^1, \widetilde{\gamma}_i^2, \widetilde{\gamma}_i^3, \widetilde{\gamma}_i^4)$

$\widetilde{b}_j^i = (b_j^{i1}, b_j^{i2}, b_j^{i3}, b_j^{i4})$

Step 2:

Using Linear Ranking function R as suggested by Rouben's, the problem (4.2) reduces to crisp multi-objective nonlinear programming problem as follows: [12, 14]

Max/ min $R(\tilde{y}) = R(\tilde{y}_1), R(\tilde{y}_2), \ldots\ldots R(\tilde{y}_p)$

Where R($(\widetilde{y}_j) = \sum_{j=1}^{n} R(\widetilde{e}_{ij}x_j^{\delta_j} + R(\widetilde{y}_j)$

Subject to $\sum R(\widetilde{a}_{jk}^i)x_j \le R(\widetilde{b}_j^i), x_k \ge 0$

This again can be written as
Max /min Y = (y_1, y_2, ….. ….. y_p)

Where $y_j = \sum_{j=1}^{n} e_{ij}x_j^{\delta_j} + \gamma_i$

. . . .(4.3)

Subject to

$$\sum a_{jk}^i x_k \le b_j^i, x_k \ge 0$$

Where e_{ij} , $\gamma_i, a_{jk}^i, b_j^i, Y, y_i, \delta_j$ are real numbers corresponding to the fuzzy numbers with respect to ranking function respectively.

Step 3:

The crisp multi-objective linear programming problem (4.3) is then solved by using fuzzy programming technique of Zimmermann. The optimal solution thus obtained shall be optimal solution of problem (4.1).

Lemma: The optimal solutions of (4.2) and (4.3) are equivalent.

Proof:

Let M_1, M_2 be sets of all feasible solutions of (4.2) and (4.3) respectively

Then, $x \in M_1$ iff $\sum_k \widetilde{a}_{jk}^i x_k \le (\widetilde{b}_j^i), i = 1,2,\ldots..m$

By considering R as a linear ranking function,

We have $\sum_k R(\widetilde{a}_{jk}^i)x_k \le R(\widetilde{b}_i)$, i = 1,2, ... m

$\Rightarrow \sum_k a_{jk}^i x_k \le b_j$

Hence, x ϵ M_2
Thus, $M_1 = M_2$

Let x*ϵ X be the complete optimal solution of (4.2)

Then, $\widetilde{y}_i(x^*) \ge \widetilde{y}_i(x)$, for all x ϵX, where X is feasible set of solutions.

$\Rightarrow R(\widetilde{y}_i(x^*)) \ge R(\widetilde{y}_i(x))$ (applying ranking function R)

$$\Rightarrow \sum_{i=1}^{n} R(\widetilde{e}_{ij})x_j^{\delta_j} + R(\widetilde{\gamma}_j) \ge \sum_{i=1}^{n} R(e_{ij})x_j^{\delta_j} + R(\widetilde{\gamma}_i)$$

$$\Rightarrow \sum_{j=1}^{n} e_{ij}x_j^{*\delta_j} + \gamma_j \ge \sum_{j=1}^{n} e_{ij}x_j^{\delta_j} + \gamma_j, \forall j = 1,2,\ldots..q$$

$$\Rightarrow \widetilde{y}_j(x^*) \ge \widetilde{y}_j(x), \forall x$$

Numerical Example:

$$\text{Max: } \left\{ \frac{\tilde{2}x_1 + \tilde{3}x_2}{\tilde{2}x_1^2}, \frac{\tilde{3}x_1 + \tilde{4}x_2}{\tilde{5}x_1^2} \right\} \ldots\ldots(5.1)$$

Subject to

$$\tilde{1}x_1 + \tilde{4}x_2 \le \tilde{4}$$
$$\tilde{1}x_1 + \tilde{1}x_2 \le \tilde{2} \quad \ldots\ldots\ldots\ldots(5.2)$$
$$x_1, x_2 \ge 0$$

Where

$\tilde{2}$ = (1.9, 2.1, 2.2, 2.6)
$\tilde{3}$ = (2.2, 2.3, 3.3, 3.8)
$\tilde{2}$ = (1.2, 2.1, 2.3, 2.8)
$\tilde{3}$ = (2.8, 2.5, 3.3, 3.8)
$\tilde{4}$ = (3.8, 3.5, 3.3, 4.8)
$\tilde{5}$ = (4.8, 4.5, 4.3, 5.4)
$\tilde{1}$ = (1.8, 0.5, 1.3, 0.9)
$\tilde{4}$ = (3.8, 4.5, 4.3, 4.8)
$\tilde{1}$ = (0.7,0.9, 1.1, 1.3)
$\tilde{1}$ = (1.8, 0.5, 1.3, 0.9)

$\tilde{4}$ = (3.3, 3.4 ,4.1, 4.4)
$\tilde{2}$ = (1.9, 2.1, 2.2, 2.6)

The MOFPP (5.1)using step 3 reduces to MOFNLPP

Max $\tilde{y}_1 = \tilde{2}x_1 + \tilde{3}x_2$ □ $\tilde{2}x_1^2$

Max $\tilde{y}_2 = \tilde{3}x_1 + \tilde{4}x_2$ □ $\tilde{5}x_1^2$

Subject to

$$\tilde{1}x_1 + \tilde{4}x_2 \le \tilde{4}$$
$$\tilde{1}x_1 + \tilde{1}x_2 \le \tilde{2} \quad \ldots\ldots(5.3)$$
$$x_1, x_2 \ge 0$$

Using Ranking Function, we have

$$y_1 = 2.2x_1 + 2.9x_2 - 1.9x_1^2 \quad \ldots\ldots (5.4)$$
$$y_2 = 2.9x_1 + 3.9x_2 - 4.9x_1^2 \quad \ldots\ldots (5.5)$$

Subject to

$$1.1x_1 + 3.9x_2 \le 3.8$$
$$1.0x_1 + 1.1x_2 \le 2. \quad \ldots\ldots (5.6)$$
$$x_1, x_2 \ge 0$$

Solving (5.4) with (5.6) by Wolf's method,

$x_1 = \frac{539}{1482} = 0.3637$, $x_2 = \frac{50387}{257798} = 0.8718$

Solving (5.5) with (5.6) by Wolf's method,

$x_1 = \frac{9}{49} = 0.18$, $x_2 = \frac{1763}{1911} = 0.9226$

The lower bound (L.B) and upper bound (U.B) of objective functions y_1 and y_2 have been computed as follows [6, 7]

Function	L.B	U.B
y_1	3.0655	3.0770
y_2	3.8065	3.9653

Now, the problem becomes[11]

Min : λ

Such that

$$2.2x_1 + 2.9x_2 - 1.9x_1^2 + 0.015 \ge 3.0770$$
$$2.9x_1 + 3.9x_2 - 4.9x_1^2 + 0.1588 \ge 3.953$$

Subject to

$$1.1x_1 + 3.9x_2 \le 3.8$$
$$1.0x_1 + 1.1x_2 \le 2.1 \quad \ldots\ldots\ldots\ldots (5.7)$$
$$x_1, x_2 \ge 0$$

Solving (5.7) the optimal solution of the problem is obtained as:

$x_1^* = 0.4518$

$x_2^* = 0.8469$

Using the above value, we get,

$\tilde{y}_1^*$ = (2.4767,2.6314,3.3194,3.8214)

$\tilde{y}_2^*$ = (2.8716,3.1109,3.9866,4.4831)

Let $\tilde{f}_1^* = \frac{\tilde{Y}_1^*}{\tilde{Y}_2^*}$

The membership function is defined as follows:

$$\mu_{\tilde{Y}_1}(x) = \begin{cases} 0 & x \le 2.4767 \\ \frac{x-2.4767}{0.1597} & 2.4767 < x \le 2.6314 \\ 1 & 2.6314 < x \le 3.3194 \\ \frac{3.3194-x}{0.502} & 3.3194 < x \le 3.8214 \\ 0 & x > 3.8214 \end{cases}$$

$$\mu_{\tilde{Y}_2}(x) = \begin{cases} 0 & x \le 2.8616 \\ \frac{x-2.8616}{0.249} & 2.8616 < x \le 3.1109 \\ 1 & 3.1109 < x \le 3.9866 \\ \frac{4.4831-x}{0.4965} & 3.9866 < x \le 4.4831 \\ 0 & x > 4.4831 \end{cases}$$

α - cuts:

$$\alpha\,\tilde{Y}_1 = [\,0.1597\alpha + 2.4767\,,\ 3.8214 - 0.502\alpha\,]$$

$$\alpha\,\tilde{Y}_2 = [\,0.249\alpha + 2.8616\,,\ 4.4831 - 0.4965\alpha\,]$$

$$\alpha\,(\tilde{Y}_1|\tilde{Y}_2) = \left(\frac{0.1597\alpha + 2.4767}{4.4831 - 0.4965\alpha},\ \frac{3.8214 - 0.502\alpha}{0.249\alpha + 2.8616}\right)$$

for $\alpha \in [0,1]$

$$\mu\tilde{f}_1^*(x) = \begin{cases} 0 & for\ \ x \le 0.2316\ and\ x \ge 0.4726 \\ \frac{4.4831x - 2.476732}{0.1597 + 0.4965x} & for\ \ 0.2316\ \ x \le 0.3438 \\ \frac{3.8214 - 2.8616x}{.502 + 0.249x} & for\ \ 0.3438\ \ x \le 0.4726 \end{cases}$$

Where $\tilde{f}_1^* = \frac{\tilde{Y}_1^*}{\tilde{Y}_2^*}$

CONCLUSION

The method adopted here for reducing the multi-objective fuzzy nonlinear fractional programming problem to that of multi-objective fuzzy nonlinear programming problem and then reducing it to crisp one using ranking function is a new approach to tackle a MOFNLFPP. In this paper, we considered fuzzy multi-objective nonlinear fractional programming problem, in which the objective function is nonlinear, but the constraints are linear. This approach can be extended to problems when both objective functions and constraints are nonlinear fractional programming problem.

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